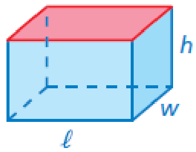
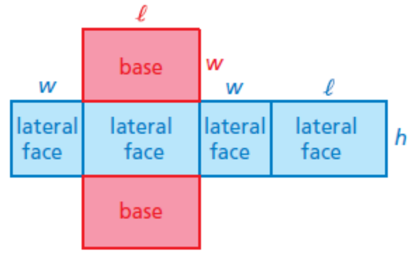
Notes 9.1	Surface Area of Prisms

Words The surface area, *SA,* of a rectangular prims is the sum of the areas of the bases and the lateral faces.





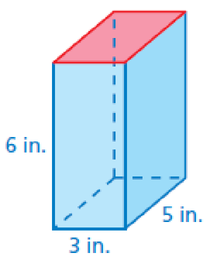
Algebra $SA = 2lw + 2lh + 2wh$

Area of bases

Area of lateral faces

Finding the Surface Area of a Prism:

Example 1: Find the surface area of the rectangular prism.



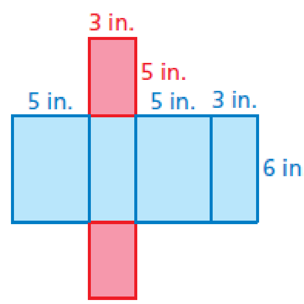
Draw a net.

$SA = 2lw + 2lh + 2wh$

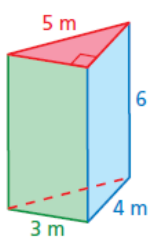
$SA = 2(3)(5) + 2(3)(6) + 2(5)(6)$

$SA = 30 + 36 + 60$

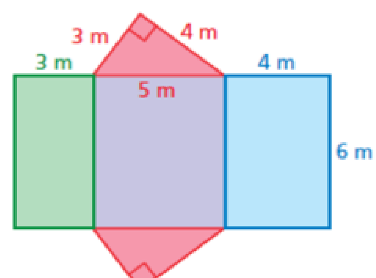
$\boxed{SA = 126 \text{ in}^2}$



Example 2: Find the surface area of the triangular prism.



Draw a net.



Area of the 2 triangular bases

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(3)(4)$ Substitute 3 in for base and 4 for the height.

$A = 6(2) = 12\ m^2$ There are 2 bases. Multiply by 2.

Area of left rectangle

$A = lw$

$A = 6(3) = 18\ m^2$

Area of middle rectangle

$A = lw$

$A = 6(5) = 30\ m^2$

Area of right rectangle

$A = lw$

$A = 6(4) = 24\ m^2$

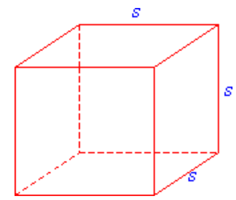
Add the areas of the bases and lateral faces (rectangles) together.

$SA = 12 + 18 + 30 + 24$

$\boxed{SA = 84 \text{ m}^2}$

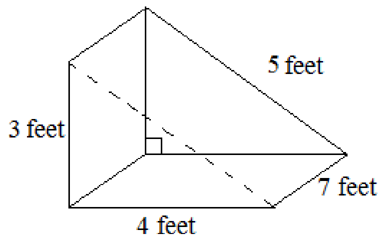
When all the edges of a rectangular prism have the same length s, the rectangular prism is a cube. The formula for the surface area of a cube is

$$SA = 6s^2$$

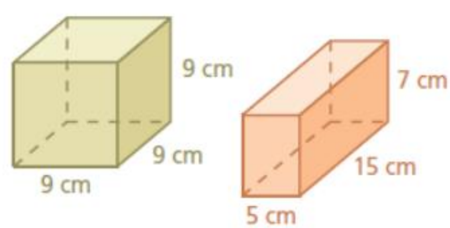


Try This: Find the surface area of the prism.

a.

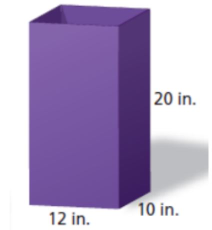


b. Which prism has the greater surface area?



The **lateral surface area** of a prism is the sum of the areas of the lateral faces.

Example 3: The outsides of fly trap are coated with glue to catch flies. You make your own trap in the shape of a rectangular prism with an open top and bottom. What is the surface area that you need to coat with glue?



Find the lateral surface area.

Do not include the areas of the bases in the formula.

$SA = 2lh + 2wh$

$SA = 2(12)(20) + 2(10)(20)$

$SA = 480 + 400$

$\boxed{SA = 880 \text{ in}^2}$

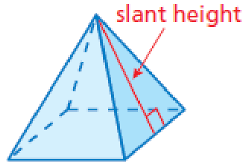
Try This:

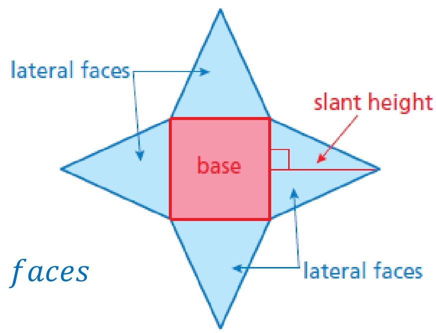
d. WHAT IF? In Example 3, both the length and the width of your trap are 12 inches. What is the surface area that you need to coat with glue?

Notes 9.2 Surface Area of Pyramids

A **regular pyramid** is a pyramid whose base is a regular polygon. The lateral faces are triangles. The height of each triangle is the **slant height** of the pyramid.

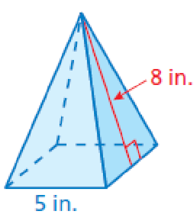
The surface area, *SA,* of a pyramid is the sum of the areas of the base and the lateral faces.





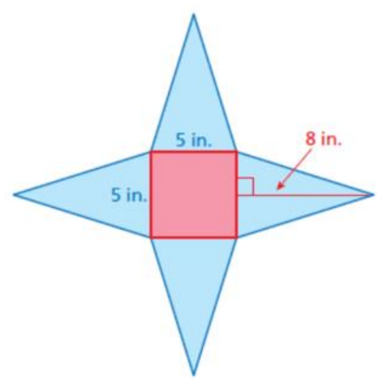
$SA = \text{area of base} + \text{areas of lateral faces}$

Example 1: Find the surface area of the regular pyramid.



Draw a net.

Area of Base	***Area of a Lateral Face***
$A = lw$	$A = \frac{1}{2}bh$
$A = 5(5)$	$A = \frac{1}{2}(5)(8)$
$A = 25\ in^2$	$A = \frac{1}{2}(40)$
	$A = 20\ (4) = 80\ in^2$ There are 4 identical lateral faces.

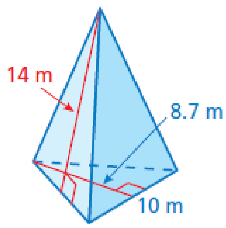


$\boxed{SA = 25 + 80 = 105\ in^2}$

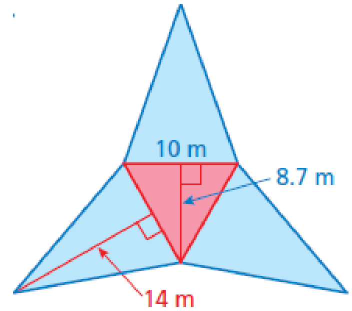
Try This:

a. What is the surface area of a square pyramid with a base side length of 9 centimeters and a slant height of 7 centimeters?

Example 2: Find the surface area of the regular pyramid.



Draw a net.



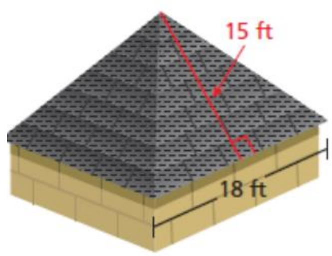
Area of Base	***Area of a Lateral Face***
$\boldsymbol{A = \frac{1}{2}bh}$	$\boldsymbol{A = \frac{1}{2}bh}$
$\boldsymbol{A = \frac{1}{2}(10)(8.7)}$	$\boldsymbol{A = \frac{1}{2}(10)(14)}$
$\boldsymbol{A = 43.5\ m^2}$	$\boldsymbol{A = 70(3) = 210\ m^2}$ There are 3 lateral faces.

Add the areas together.

$SA = 43.5 + 210$

$\boxed{SA = 253.5\ \text{m}^2}$

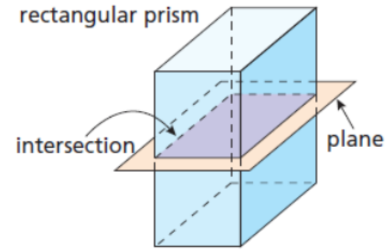
Example 3: A roof is shaped like a square pyramid. One bundle of shingles covers 25 square feet. How many bundles should you buy to cover the roof?



Notes 9.3	Cross Sections and Volume of Prisms

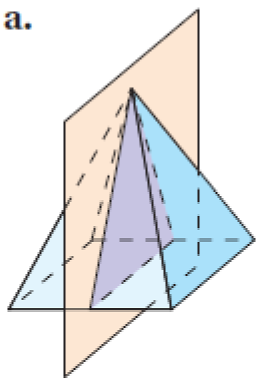
Cross Sections of Three-Dimensional Figures

Consider a plane "slicing" through a solid. The intersection of the plane and the solid is a two-dimensional shape called a **cross section.** For example, the diagram shows that the intersection of the plane and the rectangular prism is a rectangle.

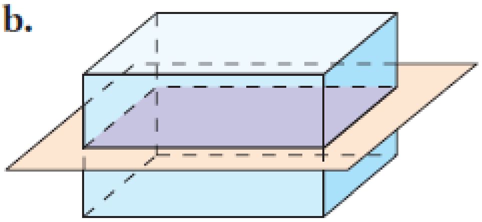


Example 1: Describe the intersection (cross section) of the plane and the solid.

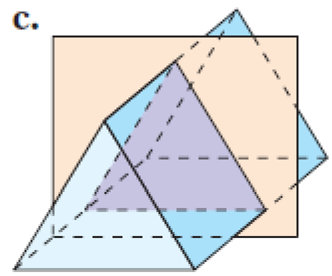
a.



b.

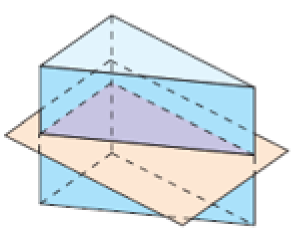


c.

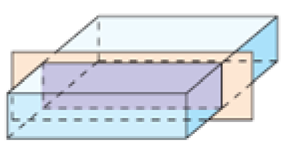


Try These: Describe the intersection (cross section) of the plane and the solid.

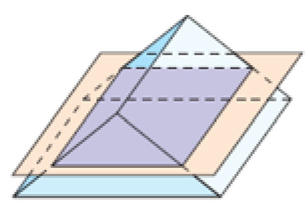
1.



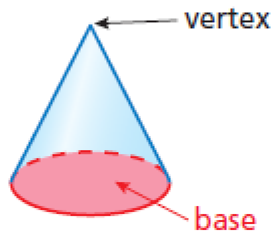
2.



3.

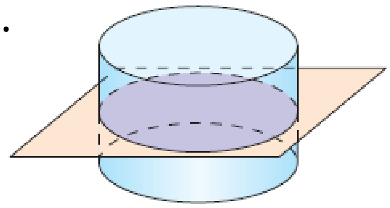


Example 1 shows how a plane intersects a polyhedron. Now consider the intersection of a plane and a solid having a curved surface, such as a cylinder or cone. As shown, a *cone* is a solid that has one cirular base and vertex.

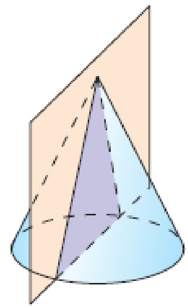


Example 2: Describe the intersection (cross section) of the plane and the solid.

a.

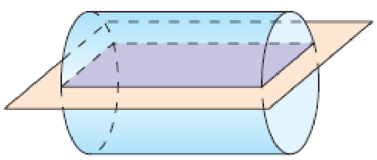


b.

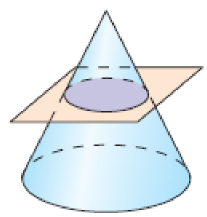


Try These: Describe the intersection (cross section) of the plane and the solid.

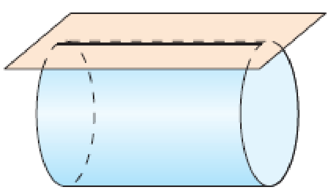
7.



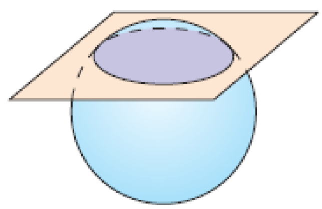
8.



9.



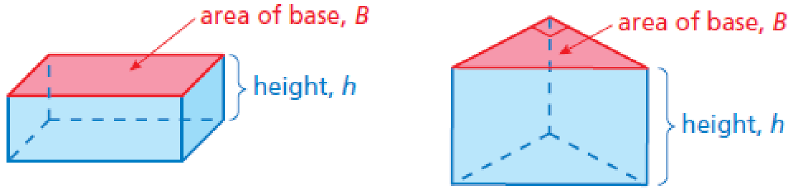
10.



Volume of Prisms

The ***volume*** of a three-dimensional figure is a measure of the amount of space that it occupies. Volume is measured in cubic units.

Words The volume, *V*, of a prism is the product of the area of the base and the height of the prism.

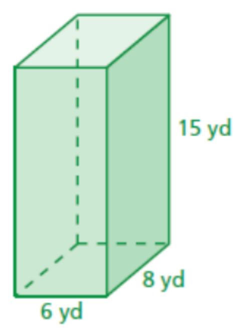


Algebra

$$V = Bh$$

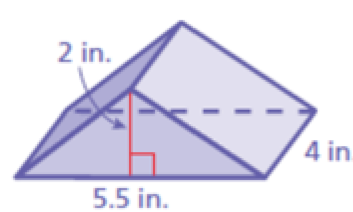
Area of base   Height of prism

Example 1: Find the volume of the rectangular prism.



$\boldsymbol{V = Bh}$	Write formula for volume.
$V = (lw)h$	Substitute *lw* for *B*.
$V = 6(8)(15)$	Substitute.
$\boxed{V = 720 \text{ yd}^3}$	Multiply.

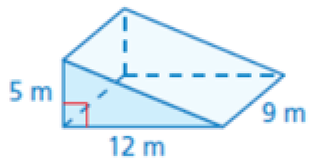
Example 2: Find the volume of the triangular prism.



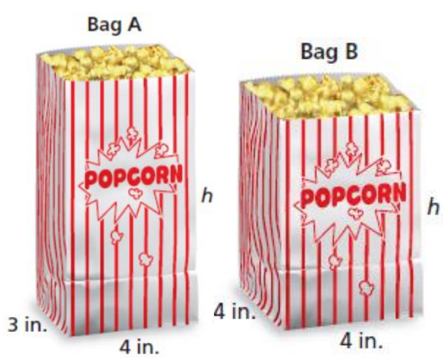
$V = Bh$	Write formula for volume. Base is a triangle.
$V = \left(\frac{1}{2}bh\right)h$	Substitute area of a triangle formula for B.
$V = (\frac{1}{2} \bullet 5.5 \bullet 2) \bullet 4$	Multiply.
$V = 5.5(4)$	Multiply.
$\boxed{V = 22 \text{ in}^3}$	

Try This: Find the volume of the prism.

a.



Example 3: A movie theater designs two bags to hold 96 cubic inches of popcorn.



(a) Find the height of each bag.

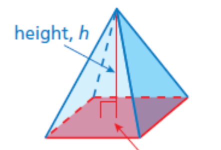
(b) Which bag should the theater choose to reduce the amount of paper needed? (*Hint: find the surface area of each bag. Remember, there is no top).*

Words The volume, *V*, of a pyramid is one-third the product of the area of the base and the height of the pyramid.

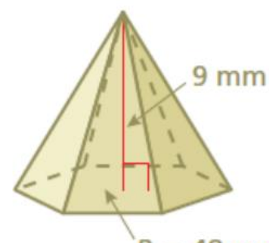
The *height* of a pyramid is the perpendicular distance from the base to the vertex.

Algebra

$$V = \frac{1}{3}\underbrace{B}_{\text{Area of base}}\underbrace{h}_{\text{Height of pyramid}}$$



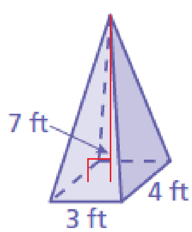
Example 1: Find the volume of the pyramid.



$V = \frac{1}{3}Bh$	**Write formula for volume.**
$V = \frac{1}{3}(48)(9)$	**Substitute 48 for B and 9 for h.**
$V = (16)(9)$	**Multiply.**
$\boxed{V = 144\ \text{mm}^3}$	

Example 2: Find the volume of each pyramid.

a.



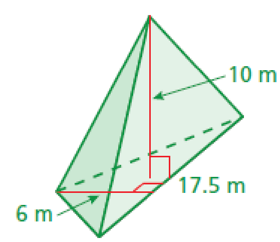
$V = \frac{1}{3}Bh$

$V = \frac{1}{3}(lw)h$

$V = \frac{1}{3}(4 \cdot 3)(7)$

$\boxed{V = 28\ feet^3}$

b.



$V = \frac{1}{3}Bh$

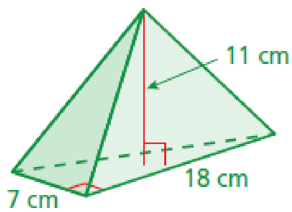
$V = \frac{1}{3}\left(\frac{1}{2}bh\right)h$

$V = \frac{1}{3}\left(\frac{1}{2} \cdot 17.5 \cdot 6\right)(10)$

$\boxed{V = 175\ meters^3}$

Try This: Find the volume of the pyramid.

a.



Example 3: Below are two different size sunscreen bottles.

a. The volume of sunscreen in Bottle B is about how many times the volume in Bottle A?

Find the volume of each bottle.

<u>**Bottle A**</u>	<u>**Bottle B**</u>
$V = \frac{1}{3}Bh$	$V = \frac{1}{3}Bh$
$V = \frac{1}{3}(lw)h$	$V = \frac{1}{3}(lw)h$
$V = \frac{1}{3}(2 \bullet 1)(6)$	$V = \frac{1}{3}(3 \bullet 1.5)(4)$
$V = 4 \text{ in}^3$	$V = 6 \text{ in}^3$

So, the volume of sunscreen in Bottle B is about $\frac{6}{4} = 1.5$ times the volume in Bottle A.

b. Find the unit price for each bottle.

<u>**Bottle A**</u>

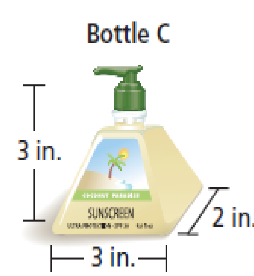
$$\frac{cost}{volume} = \frac{\$9.96}{4\ in^3} = \$2.49 \,/\, 1\ in^3$$

<u>**Bottle B**</u>

$$\frac{cost}{volume} = \frac{\$14.40}{6\ in^3} = \$2.40 \,/\, 1\ in^3$$

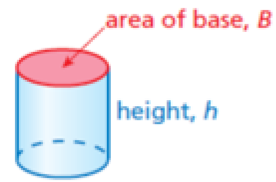
The unit cost of Bottle B is less than the unit cost of Bottle A. So, Bottle B is a better buy.

c. Bottle C is on sale for $13.20. Is Bottle C a better buy than Bottle B in Example 3? Explain.



Notes 9.5 Volume of Cylinders and Cones

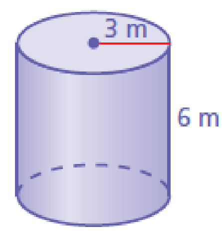
Words The volume, *V*, of a cylinder is the product of the area of the base and the height of the cylinder.



Algebra $V = Bh$

Area of base — Height of cylinder

Example 1: Find the volume of the cylinder. Round to the nearest tenth.



$V = Bh$ Write the formula for volume.

$V = (\pi r^2)h$ Substitute the area of a circle formula in for B.

$V = \pi(3)^2(6)$ Substitute 3 in for radius and 6 for height.

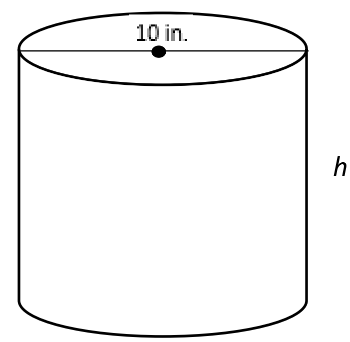
$V = 54\pi \approx 169.64$ Multiply and round.

$\boxed{V \approx 169.6\ m^3}$

Example 2: Find the height of the cylinder. Round to the nearest whole number.

The diameter is 10 inches. So, the radius is 5 inches.

$V = 314\ in^3$



$V = Bh$ Write the formula for volume.

$V = (\pi r^2)h$ Substitute the area of a circle formula in for B.

$314 = (\pi(5)^2)(h)$ Substitute 314 in for V and 5 in for the radius.

$314 = 25\pi h$ Multiply.

$\frac{314}{25} = \frac{25\pi h}{25}$ Divide both sides by 25.

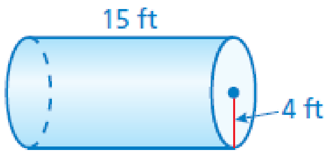
$\frac{12.56}{\pi} = \frac{\pi h}{\pi}$ Divide both sides by π.

$3.9979 \approx h$ Round.

$\boxed{h \approx 4\ inches}$

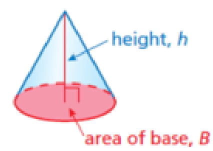
Try This:

a. Find the volume of the cylinder to the nearest tenth.

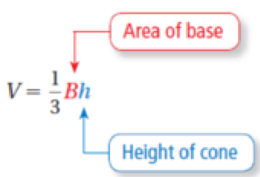


Volume of a Cone

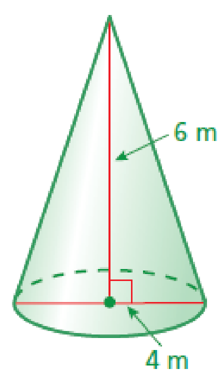
Words The volume, *V*, of a cone is one-third the product of the area of the base and the height of the cone.



Algebra



Example 3: Find the volume of the cone. Round to the nearest tenth.

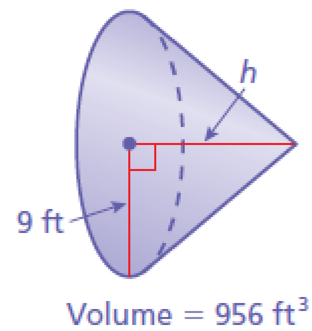


The diameter is 4 meters. So, the radius is 2 meters.

$V = \frac{1}{3}Bh$	Write formula for volume.
$V = \frac{1}{3}(\pi r^2)h$	Substitute the area of a circle formula in for B.
$V = \frac{1}{3}(\pi(2)^2)(6)$	Substitute 2 in for the radius and 6 in for the height.
$V = 8\pi$	Multiply.
$V \approx 25.132$	Round.

$$\boxed{V \approx 25.1\ m^3}$$

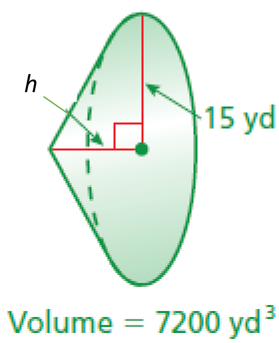
Example 4: Find the height of the cone. Round to the nearest tenth.



$V = \frac{1}{3}Bh$	Write formula for volume.
$V = \frac{1}{3}(\pi r^2)h$	Substitute the area of a circle formula in for B.
$956 = \frac{1}{3}(\pi(9)^2)(h)$	Substitute 956 for V and 9 for radius.
$956 = 27\pi h$	Multiply.
$\frac{956}{27} = \frac{27\pi h}{27}$	Divide both sides by 27.
$\frac{35.4074}{\pi} = \frac{\pi h}{\pi}$	Divide both sides by π.
$h \approx 11.27$	Round.
$\boxed{h \approx 11.3\ feet}$	

Try This:

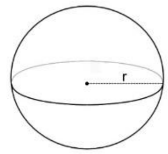
b. Find the height of the cone. Round to the nearest tenth.



Notes 9.6 Volume of Spheres

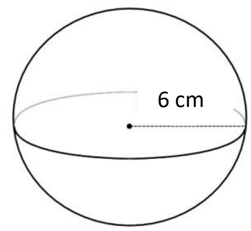
Words The volume, *V*, of a sphere is four thirds the product of π and the cube of the radius, *r*.

Symbols $V = \frac{4}{3}\pi r^3$ **Model**



You can use the formula for the volume of a sphere to solve mathematical and real-world problems.

Example 1: Find the volume of the sphere. Round to the nearest tenth.



$V = \frac{4}{3}\pi r^3$ Volume of a sphere formula.

$V = \frac{4}{3} \bullet \pi \bullet 6^3$ Substitute 6 in for r.

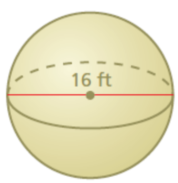
$V = \frac{4}{3} \bullet \pi \bullet (216)$ Multiply 6^3.

$V \approx 904.778$ Multiply.

$\boxed{V \approx 904.8\ cm^3}$ Round.

Try This: Find the volume of the sphere. Round to the nearest tenth.

a.



Example 2: A spherical stone in the courtyard of the National Museum of Costa Rica has a diameter of about 8 feet. Find the volume of the spherical stone. Round to the nearest tenth.

$V = \frac{4}{3}\pi r^3$ Volume of a sphere formula.

$V = \frac{4}{3} \bullet \pi \bullet 4^3$ Substitute 4 in for r.

$V = \frac{4}{3} \bullet \pi \bullet (64)$ Multiply 4^3.

$V \approx 268.0825$ Multiply.

$V \approx 268.1\ ft^3$ Round.

The volume of the spherical stone is about 268.1 cubic feet.

Try This:

b. A dish contains a spherical scoop of vanilla ice cream with a radius of 1.2 inches. What is the volume of the ice cream? Round to the nearest tenth.

Example 3: A volleyball has a diameter of 10 inches. A pump can inflate the ball at a rate of 325 cubic inches per minute. How long will it take to inflate the ball? Round to the nearest tenth.

Find the volume of the ball. Then use a proportion.

$V = \frac{4}{3}\pi r^3$	Volume of a sphere formula.
$V = \frac{4}{3} \bullet \pi \bullet 5^3$	Substitute 5 in for r.
$V = \frac{4}{3} \bullet \pi \bullet (125)$	Multiply 5^3.
$V \approx 523.59$	Multiply.
$V \approx 523.6$	Round.
$\frac{325\ in^3}{1\ min} = \frac{523.6\ in^3}{x\ min}$	Write the proportion.
$\frac{325x}{325} = \frac{523.6}{325}$	Divide both sides by 325.
$x = 1.6$	Simplify.

It will take about 1.6 minutes to inflate the ball.

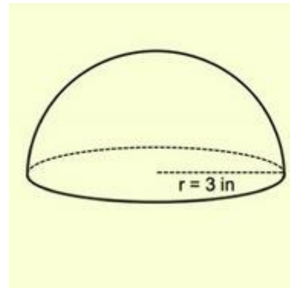
Try This:

c. A snowball has a diameter of 6 centimeters. How long would it take the snowball to melt if it melts at a rate of 1.8 cubic centimeters per minute? Round to the nearest tenth.

Volume of a Hemisphere

A circle separates a sphere into two congruent halves each called a **hemisphere.**

Example 4: Find the volume of the hemisphere. Round to the nearest tenth.



$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ Volume of a hemisphere formula.

$V = \frac{1}{2}\left(\frac{4}{3} \bullet \pi \bullet 3^3\right)$ Substitute 3 for r.

$V = \frac{1}{2}\left(\frac{4}{3} \bullet \pi \bullet 27\right)$ Multiply 3^3.

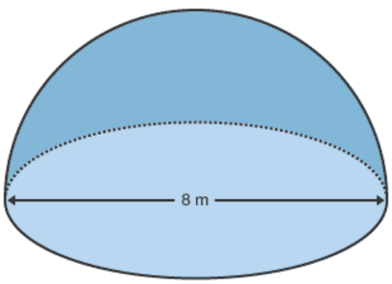
$v \approx 56.548$ Multiply.

$V \approx 56.5$ Multiply.

The volume is about 56.5 cubic inches.

Try This: Find the volume of the hemisphere. Round to the nearest tenth.

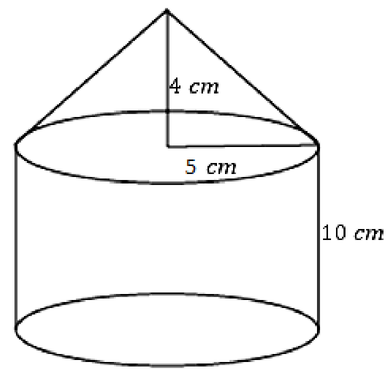
d.



Notes 9.7 Volume of Composite Solids

When a composite solid includes cylinders and cones, you can find the volume be decomposing it into solids whose volumes you know how to find.

Example 1: Find the volume of the solid. Round to the nearest tenth.



Step 1: Find the volume of the cylinder.

$V = Bh$	Volume of a cylinder formula.
$V = \pi r^2 h$	Volume of a cylinder formula.
$V = \pi \bullet 5^2 \bullet 10$	Substitute 5 in for r and 10 for h.
$V = \pi \bullet 25 \bullet 10$	Multiply 5^2.
$V = \pi \bullet 250$	Multiply.
$V \approx 785.398$	Multiply.
$V \approx 785.4\ cm^3$	Round.

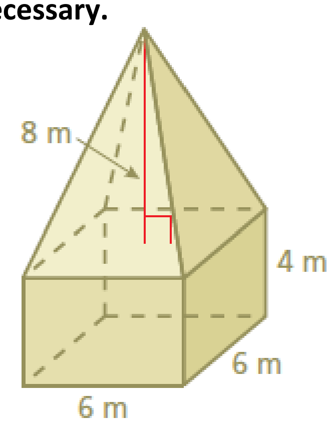
Step 2: Find the volume of the cone.

$V = \frac{1}{3}Bh$	Volume of a pyramid formula.
$V = \frac{1}{3}\pi r^2 h$	Volume of a cone formula.
$V = \frac{1}{3}(\pi \bullet 5^2 \bullet 4)$	Substitute 5 in for r and 4 for h.
$V = \frac{1}{3}(\pi \bullet 25 \bullet 4)$	Multiply 5^2.
$V = \frac{1}{3}(\pi \bullet 100)$	Multiply.
$V \approx 104.719$	Multiply.
$V \approx 104.7\ cm^3$	Round.

Step 3: Add the two volumes together: 785.4 + 104.7 = 890.1

The volume is about 890.1 cubic centimeters.

Example 2: Find the volume of the solid. Round to the nearest tenth if necessary.



Step 1: Find the volume of the pyramid.

$V = \frac{1}{3}Bh$ Volume of a pyramid formula.

$V = \frac{1}{3}(lw)h$ Base is a square.

$V = \frac{1}{3}(6 \bullet 6) \bullet 8$ Substitute 6 in for *l* and *w*. Substitute 8 in for *h*.

$V = 96\, m^3$ Multiply.

Step 2: Find the volume of the prism.

$V = Bh$ Volume of a prism formula.

$V = lwh$ Volume of a prism formula.

$V = 6 \bullet 6 \bullet 4$ Substitute 6 in for *l*, 6 in for *w*, and 4 in for *h*.

$V = 144\, m^3$ Multiply.

Step 3: Add the two volumes together: 96 + 144 = 240

The volume is 240 m^3.

Try This: Find the volume of the solid. Round to the nearest tenth if necessary.

a.